**On Characteristics of Translation and Multiplication in Doubt Fuzzy Algebra**

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**Abstract:** This paper introduces novel algebraic structures termed and within the framework of in -Algebras. It also extends the definitions of and to of -Algebras. The study explores and establishes key properties and characteristics of these structures in the context of -Algebras, offering a deeper understanding of their behaviour and interactions. The proposed notation enhances the conceptual foundation of Doubt Fuzzy Algebra and contributes to the broader field of abstract algebra by providing new tools for analysing uncertainty and fuzziness in algebraic systems.

**Keywords:** Fuzzy Set (), Fuzzy Subset (), Doubt Fuzzy Ideal (), Doubt Fuzzy Sub Algebra (), Doubt Fuzzy Translation Doubt Fuzzy Multiplication .

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**I INTRODUCTION**

The foundation of fuzzy sets (FSs) was laid by Zadeh in 1965 [14]. Later, in 1978, Tanaka [3] explored the basic framework of BCK-algebras. In 1980, Iseki [4] presented the initial formulation of BCI-algebras. The notion of fuzzy translations and fuzzy multiplication within BCK/BCI-algebras was introduced by Lee in 2009 [5]. In 2014, Ayub [1] contributed by examining fuzzy translations in the context of fuzzy β-ideals in β-algebras. That same year, Priya [11] proposed a novel approach involving fuzzy translation and multiplication in PS-algebras. Between 2018 and 2019, Ismail [6], [7] extended this line of study by focusing on fuzzy translation and multiplication in B-algebras and later in BG-algebras. In 2020 and 2021, Prem [8], [9], [10] introduced and analyzed algebraic properties related to fuzzy translation and multiplication in BH–algebras and BP–algebras, including ω-fuzzy structures and κ-Q-fuzzy concepts in T-ideals of T-algebras. Sowmi [12], [13] further expanded the research in 2019 by exploring fuzzy Z-ideals and the fuzzy algebraic framework within Z-algebras.

In this study, we introduce a novel framework for the algebraic structures known as and within the context of , a subclass of algebras. Additionally, we define and in in the setting of , another structure within -algebras. Various properties associated with these operations are examined and discussed. The newly proposed concepts aim to expand the theoretical foundation of fuzzy algebraic systems, offering deeper insights into the behavior and characteristics of -based transformations within -algebraic frameworks. These findings may contribute to further advancements in fuzzy algebra and related mathematical structures.

## II PRELIMINARIES

**Basic Concept: 1**

A -algebra ( be a -algebra. A in , is said to be a of a -algebra ,

**Basic Concept: 2**

A -algebra ( be a-algebra. A in , is said to be a of a Ž-algebra

**III** **ALGEBRAIC STRUCTURES OF TRANSLATION AND MULTIPLICATION IN DOUBT FUZZY -SUBALGEBRA**

Let, be a algebra. For any of , we define =1-

**Definition: 3**

Let, be a of and . A is said to be a of , .

**Definition: 4**

Let, be a of and . A is said to be a of , .

**Example:**

Let, = be the set

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| \* | 0 | 1 | 2 | 3 |
| 0 | 0 | 1 | 2 | 3 |
| 1 | 1 | 0 | 1 | 1 |
| 2 | 2 | 2 | 0 | 2 |
| 3 | 3 | 3 | 3 | 0 |

is a – algebra.

= 1-0.4 = 0.6,

Put and

is

⇒ is a .

is

⇒ is a

**Theorem: 1**

Let, of is a and then of is also a of .

**Proof:**

Let, and

Then,

Now,

[

=

=.

**Theorem: 2**

Let, be a of s.t of is a of Ӿ, then is a of Ӿ.

**Proof:**

Let, is a of Ӿ

Let,

=

= [

Hence, is of Ӿ.

**Theorem: 3**

Any of Ӿ and , if the of is a of Ӿ.

**Proof:**

Let and

Then ≥

Now,

[

=

is a of Ӿ.

**Theorem: 4**

Any , of Ӿ and , if the of is a of Ӿ, then so in

**Proof:**

Let, of is a of Ӿ

Let,

=

[

[

is a of Ӿ.

**IV ALGEBRAIC STRUCTURES OF TRANSLATION AND MULTIPLICATION IN DOUBT FUZZY IDEAL OF Z-ALGEBRA**

**Theorem: 5**

Let, of is a , then, .

**Proof:**

**Theorem: 6**

Let, is a of Ӿ, then of is a of Ӿ.

**Proof:**

Let, be a of Ӿ

Then,

(i)=

=

(ii)

of is a of Ӿ,

**Theorem: 7**

Let, is a of Ӿ s.t the of is a of Ӿ, then, is a of Ӿ.

**Proof:**

Given, is a of Ӿ

Let,

Then,

=

And so ⇒

and so

is a of Ӿ.

**Theorem: 8**

Let, and be a of Ӿ. If Ӿ is a Z-algebra, then of is a of Ӿ.

**Proof:**

Let,

is a of Ӿ.

**Theorem: 9**

Let, of is a of Ӿ, then is a of Ӿ.

**Proof:**

Given that, of is a of Ӿ.

Then

⇒

is a of Ӿ.

**Theorem: 10**

Let is a of Ӿ s.t the of is a of Ӿ, then is a of Ӿ.

**Proof:**

Let, is a of Ӿ

Let

=

And so ⇒

and so

is a of Ӿ.

**Theorem: 11**

If, is a of Ӿ, then of is a of Ӿ,

**Proof:**

Let, be a of Ӿ

Then

⇒

Hence, of is a of Ӿ,

**Theorem: 12**

Let, and let, be a of a Z-algebra Ӿ. Then of is a of Ӿ.

**Proof:**

Let,

Hence is a of Ӿ,

**Theorem: 13**

If the of is a of Ӿ, then is a of Ӿ.

**Proof:**

Let of is a of Ӿ.

Then

⇒

Hence is a of Ӿ.

**Theorem: 14**

Let and of any 2 T of a of of Ӿ is also a of Ӿ.

**Proof:**

Let, and be two T of a of of Ӿ,

Assume that

and are of Ӿ.

∩

And =

∩ and are of Ӿ.

**V CONCLUSION**

This paper explores the concepts of ύ-Ṱ and ύ-Ӎ within the framework of Ž-Algebras, focusing specifically on their behavior in the context of DF-ŽSA. Various related properties have been examined and analyzed in detail. Additionally, the study delves into the derivation of ύ-Ṱ and ύ-Ӎ on the DF-ŽI structure within DF-ŽA. The investigation provides insights into the algebraic interactions and theoretical underpinnings of these elements. By establishing connections between these constructs, the paper contributes to a deeper understanding of their roles and relationships in the broader landscape of Ž-Algebraic systems.

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